

TACHYON DEFECT FORMATION AND REHEATING IN BRANE-ANTIBRANE INFLATION

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We study analytically the dynamical formation of lower dimensional branes at the endpoint of brane-antibrane inflation through the condensation of topological defects of the tachyon field which describes the instability of the initial state. We then use this information to quantify the efficiency of the reheating which is due to the coupling of time dependent tachyon background to massless gauge fields which will be localized on the final state branes. We improve upon previous estimates indicating that this can be an efficient reheating mechanism for observers on the brane.

1. Brane-Antibrane Inflation and the Model of Reheating

We begin by briefly describing brane-antibrane inflation and the scenario we have in mind for reheating, see ¹ for a more detailed discussion and references. In the simplest model of D-brane inflation a parallel brane and antibrane begin with some separation between them in one of the extra dimensions. There is an attractive force between the branes, primarily due to the exchange of massless bulk modes. The branes move towards each other with the inter-brane separation playing the role of the inflaton and the non-vanishing potential energy between the branes driving inflation. Once the branes have reached a critical separation one of the stretched string modes between the branes, T , becomes tachyonic which provides a natural mechanism to end inflation since the brane-antibrane pair then become unstable to annihilate. The annihilation of the initial state brane-antibrane pair is

**presenter*

described by the rolling of the tachyon field from the unstable maximum of its potential to the degenerate vacua. The tachyon field may form topological defects through the Kibble mechanism so that $T = 0$ stays fixed at the core of the defect. These defects are known to be consistent descriptions of lower dimensional branes and hence the condensation of tachyon defects describes the formation of lower dimensional branes at the endpoint of brane-antibrane inflation.

Our interest in studying the endpoint of D-brane inflation is motivated by questions about the efficiency of the reheating. It is possible that the energy density liberated from the brane collisions will be converted into closed string states (ultimately gravitons) and not necessarily into visible radiation. The formation of tachyon defects at then endpoint of D-brane inflation is a dynamical process where the tachyon couples to gauge fields which will be localised on the descendant brane. It is thus expected that some radiation will be produced by the rolling of the tachyon and the problem of reheating becomes quantitative: can this effect be efficient enough to strongly deplete the energy density of the tachyon fluid so that the universe starts out dominated by radiation rather than cold dark matter? This question has been addressed in a similar context in ².

To study the reheating quantitatively we consider a model in which our universe is a D3-brane which is the decay product of the annihilation of a D5-D $\bar{5}$ pair. We will begin by studying tachyon defect formation in an effective 6D theory with the two extra dimensions compactified on a sphere. In this picture our universe is a tachyon vortex located at the north pole of the 2-sphere. Conservation of RR charge requires that a D $\bar{3}$ -brane also be created in the annihilation which we place at the south pole of the 2-sphere, to preserve azimuthal symmetry. Throughout the analysis we will study the formation of only one defect since both defects are identical and the solutions may be matched along the hemisphere. Furthermore, since the defects will be highly localized at the poles we will ignore the compact topology of the extra dimensions in solving for the vortex background.

2. Effective Field Theory on the Brane-Antibrane Pair

We will work with an effective field theory proposed by Sen ³ for the tachyon on the brane-antibrane pair. The field content for this system is a complex tachyon field T , massless gauge fields $A_M^{(1)}$, $A_M^{(2)}$ ^a and scalar fields $Y_{(1)}^I$,

^aUpper case roman indices $\{M, N\}$ run over the full space-time coordinates $\{0, 1, \dots, p\}$. We also define greek indices $\{\mu, \nu\}$ which run over the coordinates $\{0, 1, 2\}$ on which

$Y_{(2)}^I$ corresponding to the transverse fluctuations of the branes. The index $(i) = (1), (2)$ labels which of the original branes (actually the brane or the antibrane) the field is associated with. The effective action is:

$$S = - \int V(T, Y_{(1)}^I - Y_{(2)}^I) \left(\sqrt{-\det M^{(1)}} + \sqrt{-\det M^{(2)}} \right) d^{p+1}x \quad (1)$$

where

$$M_{MN}^{(i)} = g_{MN} + \alpha' F_{MN}^{(i)} + \partial_M Y_{(i)}^I \partial_N Y_{(i)}^I + \frac{1}{2} D_M T D_N T^* + \frac{1}{2} D_M T^* D_N T,$$

$$F_{MN}^{(i)} = \partial_M A_N^{(i)} - \partial_N A_M^{(i)}, \quad D_M = \partial_M - iA_M^{(1)} + iA_M^{(2)}.$$

For the remainder of this paper we will ignore the transverse scalars and choose $V(T, 0) = V(T) = \tau_p \exp(-|T|^2/a^2)$ where a is chosen so that the static singular vortex solutions of the theory (1) have the correct tension to be interpreted as codimension 2 D-branes according to the normalization proposed in ³.

As a first step we would like to study the dynamics of formation of a tachyon vortex without considering the coupling to the photon. To construct vortex solutions we ignore the transverse scalars and take the remaining fields to depend only on the polar coordinates $x^\mu = (x^0, x^1, x^2) = (t, r, \theta)$ with metric $g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dr^2 + r^2 d\theta^2$ ^b. Since the vortex solution should have azimuthal symmetry we make the ansatz:

$$T(t, r, \theta) = e^{i\theta} f(t, r), \quad A_\theta^{(1)} = -A_\theta^{(2)} = \frac{1}{2} g(t, r) \quad (2)$$

with all other components of $A_M^{(i)}$ vanishing.

2.1. Solutions Near the Core of the Defect

To analytically study the dynamics near the core of the vortex, $r = 0$, we make the ansatz:

$$f(t, r) \cong p(t)r, \quad g(t, r) \cong q(t)r \quad (3)$$

for small r . Plugging (3) into the equations of motion which follow from (1) and dropping terms which are subleading in r yields a set of coupled

the vortex solutions depend on hatted greek indices $\{\hat{\mu}, \hat{\nu}\}$ which run over the spatial coordinates parallel to the vortex $\{2, 3, \dots, p\}$. Here $p = 6$ for a vortex which describes a 3-brane.

^bRecall that we are ignoring the compact topology of the extra dimensions. The radial coordinate r measures distance along the surface of the 2-sphere from the north/south pole for the vortex/antivortex solution.

ODEs for $p(t)$ and $q(t)$. One finds that both $p(t)$ and $q(t)$ are singular in finite time t_c , and that near t_c we can approximate $p(t)$ and $q(t)$ by

$$p(t) = \frac{p_0}{t_c - t}, \quad q(t) = \frac{q_0}{t_c - t}. \quad (4)$$

2.2. Vacuum Solutions

Away from the core of the defect the fields $f(t, r)$ and $g(t, r)$ tend towards their vacuum values $f \rightarrow \infty$ and $g \rightarrow 1$. To study the dynamics in this regime we make the ansatz

$$g(t, r) = 1 - \varepsilon \sigma(t, r)$$

and work only to leading order in ε . One finds that the tachyon field must obey the eikonal equation: $-(\partial_t f)^2 + (\partial_r f)^2 + 1 = 0$. To circumvent the problem caustic formation in the bulk⁴ we take the simple solution $f(t, r) = t$. Given this solution any function $\sigma(t, r) = \sigma(r)$ satisfying the necessary boundary condition $\sigma(r \rightarrow \infty) \rightarrow 0$ will generate a solution.

2.3. Stress-Energy Tensor

The finite time divergence in the slope of the tachyon field (4) will lead to the finite time formation of a singularity in the stress energy tensor of the vortex. This divergence has the form of a delta function in which $t_c - t$ plays the role of the small parameter which regularizes the delta function. In the limit as $t \rightarrow t_c$ one finds that the energy momentum tensor for the vortex is identical to that of a $D(p-2)$ -brane

$$\begin{aligned} T^{00} &= \tau_{p-2} \delta(r \cos \theta) \delta(r \sin \theta) + 2r \Sigma(r) \\ T^{11} &= T^{22} = 0 \\ T^{\hat{\mu}\hat{\nu}} &= -\delta^{\hat{\mu}\hat{\nu}} \tau_{p-2} \delta(r \cos \theta) \delta(r \sin \theta) \end{aligned} \quad (5)$$

where $\Sigma(r)$ is subject to mild constraints from conservation of energy but is otherwise arbitrary. The extra bulk energy density $2r \Sigma(r)$ corresponds to tachyon matter rolling in the bulk.

3. Inclusion of Massless Gauge Fields

We consider now the coupling of massless gauge fields to the time dependent tachyon vortex background described above. There are two gauge fields in the problem: $A_{(1)}^M$ and $A_{(2)}^M$, or equivalently $A_+^M = A_{(1)}^M + A_{(2)}^M$ and $A_-^M = A_{(1)}^M - A_{(2)}^M$, which have different couplings to the tachyon. We have

already shown that A_-^μ is the field which condenses in the vortex, hence its associated gauge symmetry is spontaneously broken. For reheating it is thus $A_+^{\hat{\mu}}$ which most closely resembles the Standard Model photon. We will ignore fluctuations of the heavy fields A_-^μ , A_+^μ , and A_-^μ , keeping only the background solution for A_-^μ (which was given in section 2), and the fluctuations of the photon $A_+^{\hat{\mu}}$. To compute the production of photons in the time-dependent background, we want to expand the action (1) to quadratic order in $A_+^{\hat{\mu}}$. The result is

$$S = -\frac{\alpha'^2}{4} \int V(f) \sqrt{-G} G^{MN} \delta^{\hat{\mu}\hat{\nu}} \partial_M A_{\hat{\mu}}^+ \partial_N A_{\hat{\nu}}^+ d^{p+1}x \quad (6)$$

where the “tachyon effective metric” G_{MN} can be related to the vortex stress-energy tensor as $rT^{MN} = -2V(f)\sqrt{-G}G^{MN}$. From (6) one sees that the fluctuations of the photon behave like a collection of massless scalar fields propagating in a nonflat spacetime described by the metric G_{MN} , with a position- and time-dependent gauge coupling given by $g^2 = 1/V(f(t, r))$.

In the limit of condensation, T^{MN} is given by (5), so that once the brane has formed the action (6) reduces to a description of gauge fields propagating in a (3+1)-dimensional Minkowski space, with an additional component which couples the gauge fields to the tachyon matter density in the bulk. In other words, the effective metric G_{MN} starts off being smooth throughout the bulk, but within the time t_c , its support collapses to become a delta function $\delta^{(2)}(\vec{x})$ in the relevant extra dimensions $\{r, \theta\}$.

4. Simplified Model of Reheating

Notice that the extra bulk energy density $2r\Sigma(r)$ in (5) yields a coupling between the gauge field $A_{\hat{\mu}}^+$ and the tachyon matter rolling in the bulk. By changing coordinates which are comoving with the contraction of the vortex one can show that this coupling can be ignored in (6) since the gauge field is confined to the descendant brane. The situation then closely resembles a gauge theory defined on a manifold in which a two-dimensional subspace which is shrinking with time. As a simplified model of the interaction we thus consider a massless spin-1 field

$$S = -\frac{1}{4} \int \sqrt{-g} g^{MA} g^{NB} F_{MN}^+ F_{AB}^+ d^{p+1}x \quad (7)$$

propagating in a FRW-like background

$$g_{MN} dx^M dx^N = -dt^2 + R(t)^2 (d\tilde{r}^2 + \tilde{r}^2 d\theta^2) + \delta_{\hat{\mu}\hat{\nu}} dx^{\hat{\mu}} dx^{\hat{\nu}}. \quad (8)$$

The coordinate \tilde{r} in (8) is fixed with the expansion.

We will impose homogeneous boundary conditions at $\tilde{r} = 1$ and take the scale factor in (8) to be

$$R(t) = \begin{cases} R_0 & \text{if } t < 0; \\ R_0 - t & \text{if } 0 \leq t \leq t_c; \\ R_0 - t_c = \epsilon & \text{if } t > t_c \end{cases}$$

where R_0 represents the initial radial size of the extra dimensions and ϵ represents the final state brane thickness. A shortcoming of this approximation is that $\partial_t R$ is discontinuous at the interfaces, which leads to an ultraviolet divergence in the production of gauge bosons. The behavior of the actual $R(t)$ is smooth, and must yield a finite amount of particle production² so we introduce a UV cut-off $p_{max} = \Lambda$ which we will assume is of order the inverse string length l_s^{-1} . In addition, we will find a separate UV divergence in the particle production in the limit as the vortex core thickness goes to zero ($\epsilon \rightarrow 0$). This is presumably an artifact of the effective field theory which is not present in the full string theory, and we deal with it by introducing the cutoff ϵ on the final radius of the defect core, which we will take to be of order l_s .

The equations of motion which follow from (7) can be solved exactly in each of the three regions of the spacetime $t < 0$, $0 \leq t \leq t_c$ and $t > t_c$. In the asymptotic regions $t < 0$ and $t > t_c$ the gauge field is just the usual sum of plane waves with annihilation/creation operators $\{a_{mn}, a_{mn}^\dagger\}$ and $\{d_{mn}, d_{mn}^\dagger\}$ respectively. The indices $\{m, n\}$ label the KK modes of the photon. The mass spectrum in the final state is of order $\epsilon^{-1} \sim l_s^{-1}$ which is the same order as the UV cut-off so we are obligated to neglect the massive modes for consistency.

5. Energy Density of Produced Radiation

We assume that the universe starts out in the vacuum which is annihilated by a_{mn} , $a_{mn}|0\rangle = 0$. However, this vacuum is not annihilated by d_{mn} which is a linear superposition of a_{mn} and a_{mn}^\dagger . The coefficients in this superposition are the Bogoliubov coefficients which are determined by matching the gauge field solutions in each of the three regions of the spacetime. The spectrum of particles observers in the future will see is then given by

$$N_{mn}(p) = \langle 0 | d_{mn}^\dagger(p) d_{mn}(p) | 0 \rangle$$

and the energy density of the produced radiation is

$$\rho = 2 \int \frac{d^3 p}{(2\pi)^3} p N_{00}(p)$$

where we have neglected all modes but the massless zero mode and the multiplicative factor of 2 corresponds to the two polarizations of the photon. For large extra dimensions ($R_0 \gg l_s$) we find that the energy density takes the very simple form

$$\lim_{R_0 \rightarrow \infty} \rho = \frac{1}{8\pi^2} \frac{\Lambda^2}{\epsilon^2}.$$

6. Efficiency of Reheating

To quantify the efficiency of the reheating we need to determine how much energy is available to produce the photons on the final-state 3-brane. Initially the system consisted of a D5-brane plus antibrane, whose 3D energy density was given by $2\tau_5 V_2$, where τ_5 is the tension of a D5-brane and V_2 is the volume of the compact 2-space $\{r, \theta\}$ wrapped by the branes. The final state consists of a D3-brane/antibrane with total tension $2\tau_3$. Since reheating on each final-state brane should be equally efficient, the 3D energy density available for reheating on one them, which we call the critical energy density ρ_c , is half the difference between the initial and final tensions of the branes and antibranes: $\rho_c \equiv \tau_5 V_2 - \tau_3$. In the regime where $R_0 \gg l_s$ this simplifies to $l_s^4 \rho_c = (16\pi^3 \alpha(M_s))^{-1}$ where $\alpha(M_s) \approx 1/25$ ⁵ is the gauge coupling evaluated at the string scale. We find that the criterion for efficient reheating becomes

$$\sqrt{\frac{\epsilon}{\Lambda}} \lesssim (2\pi\alpha(M_s))^{1/4} l_s \cong 0.7 l_s.$$

We find, then, that the reheating can be efficient taking Λ^{-1} , ϵ both close to the string length.

7. Conclusions and Discussion

We have argued that our visible universe might be a codimension-two brane left over from annihilation of a D5-brane/antibrane pair at the end of inflation. In this picture, reheating is due to production of standard model particles (*e.g.* photons) on the final branes, driven by their couplings to the tachyon field which encodes the instability of the initial state as well as the vortex which represents the final brane. We find that reheating can

be efficient, in the sense that a sizable fraction of the energy available from the unstable vacuum can be converted into visible radiation, and not just gravitons.

The efficiency of reheating is greatest if the radius of compactification of the extra dimensions is larger than the string length l_s . The efficiency also depends on phenomenological parameters we had to introduce by hand in order to cut off ultraviolet divergences in the calculated particle production rate: namely ϵ , a nonvanishing radius for the final brane, and Λ , an explicit cutoff on the momentum of the photons produced. The latter must be introduced to correct for discontinuous time derivatives in our simplified model of the background tachyon condensate; the actual behavior of the condensate corresponds to a cutoff of order $\Lambda \sim 1/l_s$. It is less obvious why the effective field theory treatment should give divergent results as the thickness of the final brane (2ϵ) goes to zero, but it seems clear that a fully string-theoretic computation would give no such divergence, and therefore it is reasonable to cut off the field-theory divergence at $\epsilon \sim l_s$. Given only these mild assumptions, our estimates predict that the fraction of the available energy which is converted into visible radiation is $\rho/\rho_c \cong \pi\alpha(M_s) \cong 0.25$. This simple estimate counts only photons; in a more realistic calculation, it would be enhanced by the number of light degrees of freedom which couple to the tachyon, which could be much greater than 1. Moreover it could also be enhanced by the production of massive KK modes.

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